A Theory for Networks of Power: Coalition Formation on Networks

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Abstract

Political coalitions are usually thought to form along common interests or shared political views. But especially in authoritarian regimes, personal connections appear to play an equal, if not more important role. Connections are likely more relevant in such a low-trust environment, in particular when coordinating coup conspirators or opposition activists, or other situations in which either actors, connections, or activities along those connections (i.e. side payments) need to remain secret. Expanding on the selectorate theory, this paper presents a theory in which the leader - and a possible competitor in the 2nd model - form their coalition along network ties connecting members of the selectorate. The agent-based model shows that in all but the regular (i.e. complete Erdös-Rényi random) network, hierarchies quickly emerge, which can be identified through network centrality measures. Individuals with high closeness centrality, for instance, are popular coalition partners and therefore more likely to be included in the winning coalition. Coalition leaders with high betweenness centrality, on the other hand, are most likely to fend off all possible competitors, and maintain a stable rule. Networks resembling complex patronage networks (preferential-attachment networks) have a clearer hierarchy than random networks, and therefore (in the absence of changes in the network) also produce more stable leadership.
1 Introduction

Dictators do not rule alone. They require a group of people - the inner circle - that helps them maintain stability, implement policies, collect revenues, etc. How does this “coalition of the willing” form? In their selectorate theory, Bueno de Mesquita et al. (2003) develop a model to explain the size of what they call the “winning coalition”, but not its membership. So what could help us explain who rises to the top in an authoritarian regime?

Authoritarian elites plainly form coalitions to stay in power or to topple current incumbents like their democratic counterparts do - as frequent talk of factions in almost any authoritarian regime attests. Like any coalition, those factions serve to advance the interest of their members, but those interests can rarely be neatly summarized in policy positions as is the case for at least some coalitions in a democratic context.

Common policy goals clearly play a role, but frequent mentioning of “rightists” or “leftists” does not mask the fact that personal differences often split groups of seemingly like-minded individuals, that individuals change their stated policy position according to political expediency and who they plan to associated themselves with, and that their statements are usually opaque and biased towards the official party line. These transparency issues exist in democracies as well, but the authoritarian context provides even less structures and incentives for elites to openly confess their true policy preferences. Being hard to measure directly, policy positions are thus empirically less useful in explaining coalition formation in non-democratic regimes.

The usual mathematical considerations could be expected to play a role. But the threshold needed to beat competing coalitions tends to be a lot less clear than in the coalition formation process in a parliament, which presumably makes precise strategic coalition building a lot more difficult.

However, a wealth of qualitative evidence suggests that connections play an important role in determining who enters an authoritarian regime’s inner circle. In this paper, I thus propose a theory of coalition formation along the ties of a trust network and develop an agent-based model based on it.

What are the circumstances in which network ties play a predominant role and thus
my theory is most applicable? The situation of coup conspirators comes to mind, or that of revolutionaries facing a strong surveillance regime: in these circumstances, recruiting anyone - no matter how fervent a regime critic or how valuable an asset - can expose the undertaking if the potential co-conspirator turns out to be less reliable or supportive of the cause than expected. Contenders are thus likely to include in their coalition only individuals they know and trust personally, or for whom someone they know or trust can vouch.

The case for the need to coordinate along trust ties is admittedly weaker once the leader has gained power, that is for the incumbent’s coalition. At first glance, secrecy and thus trust does not appear to be necessary for the “legitimate” government. But in authoritarian regimes, as predicted by the selectorate theory, rulers maintain their position mainly by providing their supporters with private goods. As there is no external enforcer to this contract, the parties might again only be willing to coordinate with individuals they trust.

Furthermore, the ruler usually faces a simultaneous principal-agent problem: members of the winning coalition receive those private goods not only as (monetary) resources, but also in the form of appointments to high-level positions. This helps the winning coalition members fulfill their task of warding off opposition coalitions (e.g. appointments to the security apparatus) while providing them with lucrative prebends. But those resources and positions also give their holders additional opportunities to usurp power themselves; and assigning official positions to overtly greedy or incompetent coalition members can put the ruler at danger of popular rebellion. Finally, even if both coups and popular rebellion appear unlikely, the elite might be worried of a “Gorbachev” emerging from their midst, as allegedly the Communist Party in China is (Nathan and Gilley, 2003).

These are all reasons for why a leader might prefer elites who are known and trusted not to belong to the “bad bunch” - be it defined as corrupt, overtly ambitious, or reformist - over less familiar ones.

The model presented in that paper - an agent-based model on random networks - explores the consequences of a system in which coalition partners are found along personal ties of trust. While the importance of patronage ties has been explored empirically both in qualitative and quantitative studies in the context of China (Shih et al., 2010, 2012;
Jia et al., 2013; Zhang, 2009; Choi, 2012) and the Soviet Union (Willerton, 1992), the underlying theory is limited to two basic claims: (1) being connected to a patron provides benefits like advancement in the hierarchy (Shih et al., 2012; Zhang, 2009; Choi, 2012); and (2) a patron’s strength can be determined by the number of clients (Shih et al., 2010; Willerton, 1987).

One shortcoming is thus the necessity to identify the relevant patrons in advance, which is often highly contested even among experts with insider knowledge, especially for patrons that don’t hold any official positions. In my model, I demonstrate the advantage of a SNA (social network analysis) approach, and show that knowing the network structure is enough to predict who is more likely to end up in the inner circle: individuals with relatively high closeness centrality.

A high betweenness centrality marks coalition leaders that can fend off most or all competing competitions. If the other members of the network are aware of this, then those leaders should be able to rule relatively unchallenged even without holding official positions that grant them control over the state’s monopoly of power. In short: they can be exactly those kind of behind-the-scenes leaders that patrons are often thought to be.

The model thus also provides a possible answer to the question about the power source of those patrons behind the scene. A dynamic extension of the model might even start addressing the question of causality: do patrons have a lot of connections and hold a specific position in the network because the external power (resources) they command make them attractive connections? Or is it their connections that provide them with the power to rule?

Finally, I also illustrate how the network approach produces a more nuanced picture of elite relationships than the “is/is not connected” dichotomy proposed by the studies mentioned above: individuals in my model could be connected to potential patrons through intermediaries instead of directly - in effect being closer or farther away from the patron - or connected to more than one such potential patron, for instance. There are even common network types - preferential attachment networks - that resemble the complex patronage structures proposed by very early research on patronage (Scott, 1972).

The following section reviews the network literature and concepts relevant to coalition formation. Section 3 discusses the proposed agent-based model in more detail, focusing
on coalition formation with one leader first. In section 4 I present results that explain how all networks except a few special cases create hierarchies in this case (4.1) and how an individual’s network position influence his or her chance of entering the winning coalition (4.2). Section 5 explains the model with two competitors, while section 6 presents its main results: the emergence of a dominating leader is possible (6.1), in particular in networks resembling complex patronage networks (6.2), and those dominating leaders tend to have a high betweenness centrality (6.3). Section 7 summarizes the main findings and explains their relevance to the wider literature.

2 Literature Review

2.1 Coalition Formation on Networks

The literature on authoritarian regimes has not theorized coalition formation using SNA, and has limited itself to the assertion that being connected to a patron increases the chances of being included in his or her coalition. The network literature, on the other hand, has developed theories of coalition formation and concepts relevant to that topic, but not necessarily for the context of authoritarian regimes. The goal of this paper is thus to develop a model that would fit the specific data I examine in my dissertation: an informal, directed network among Chinese elites, which changes over thirty years mainly because of entry and exit of nodes. The model should help predict who will enter the inner circle of the regime (the Politburo) and understand the role of patrons in that process. As the number of potentially relevant elites is - as in many other authoritarian regimes - quite large (more than 200 individuals at any given time), I have decided to follow the likes of Siegel (2009) and use agent-based modeling and simulations instead of attempting to find closed-form solutions in a smaller network like Perez-Oviedo (2012) does. While previous SNA research is thus relevant for my model, none of it provides a ready-made solution for this particular setting, as the following review will show.

[this literature review is incomplete - any hints on other relevant papers are greatly appreciated]

One relevant concept is that of signed graphs and structural balance (Easley and
Kleinberg, 2010, chapter 5): the adage that an enemy’s enemy is a (at least temporary) friend is certainly often mentioned in the narratives of dictators and their inner circle, and negative ties play an important role in those stories. One might thus expect that coalitions form among individuals connected by positive ties, and negative ties with the same third parties. Unfortunately, negative ties are often much harder to measure reliably, and the dataset I am currently using in my research does not contain such ties. I therefore only explore coalition formation on binary networks.

Another relevant concept is that of cliques - after all, the term itself already appears regularly in those narrative, used to describe a group of densely connected people. However, while such dense clusters certainly facilitate collective action (Chwe, 2000) in the case of palace coups, a coalition able to maintain power likely requires quite a different, more open structure, as I will show later. The selectorate theory developed in the research of authoritarian regimes, which I take as the basis for my model, operates on the assumption that actors are strategic and anticipate the actions of others. Therefore, members of a dense, but small clique should anticipate that while their conspiracy might remain undetected, they are unlikely to remain in power afterwards, as they lack the support of (i.e. ties to) the wider elite. The selectorate theory is also based on the idea of (competing) leaders, while the definition of k-cliques is based on the decentralized notion that every member has to be connected to at least k other members. I have therefore decided to build a model of coalition formation that focuses more on those leaders.

The network theories that explicitly deal with coalition formation usually do not form coalition along ties: they deal with exchange networks and explore under which circumstances nodes in weak positions unite to challenge nodes in central positions (Bonacich, 2000; Simpson and Macy, 2001; Cook and Gillmore, 1984). Informal networks among authoritarian elites are often thought to be patronage networks, which can contain an element of exchange (Scott, 1972): the client may receive scarce resources in exchange for supporting the patron, for instance. In the context of Chinese elites, however, the ties, while rarely equal, often imply mutual obligations, especially as the client’s wrong-doing can fall back on the patron: recent corruption scandals, for example, often started with the investigation of clients (see the case of Bo Xilai and Zhou Yongkang). Furthermore, this literature would predict that coalitions form between (possibly not-connected) clients.
against their patron because of their common interest, which is not what experts observe in the case of Chinese elite politics. Instead, coalitions appear to form between patrons and their clients, with patrons trying to introduce their protégés into the inner circle and appoint them as their successors.

Most relevant to the situation thus appears to be Kittel and Luhan (2013)’s experiment on coalition formation along communication networks. Theirs is a coordination game in which committee members try to agree on the share of a reward to transfer to a second group in order not to lose the remainder of the reward. In the selectorate model, the situation is more competitive: a leader needs to recruit a (possibly majority) winning coalition, whose members then receive private transfers in exchange for continuing support to the him or her.\footnote{It could of course be argued that the failure to find such a coalition carries the risk of revealing elite disagreement to the masses and raise the possibility of open elite struggle or a revolution, which could be a similar punishment as the total loss of reward in the experiment.}

Kittel and Luhan (2013) find that in a star network, the central individual serves as a focal point, whose proposal can determine the outcome everyone agrees to. It is unclear how that finding of the central actor’s importance will scale up to more complex networks. A simple star network only has two discernible positions, and the center of a star is the most central individual according to all common centrality measures (Freeman, 1979). The next section therefore discusses different centrality measures and explains why they could be relevant in a model with a more complex network.

2.2 Centrality measures and their relevance

Except in complete or empty networks, most individual’s positions are unique, in a sense that each is connected to specific set of individuals, may even have a unique number of direct connections that no (or only a few) other individuals have, etc. Centrality measures are one of the many ways to summarize positions and make them comparable within and across networks. Network centrality in particular has been associated both in theory and empirical studies with informal power (Brass and Krackhardt, 2012).

Research on connections and ascendance to the inner circle of an authoritarian regime has so far only distinguished between individuals that share a connection to a special
individual - the patron - and those that don’t. This measure thus relies both on individual characteristics and (to a limited degree) network structure: the individuals have the binary characteristic of patron/non-patron, and their positions are assigned to the binary measure of shares / does not share tie with a patron.

Centrality measures, on the other hand, usually rely on the network structure alone. There exists a multitude of centrality measures in the literature, but the following three seem most relevant to my coalition formation model:

Degree measures each position according to the number of other position it is connected to, in other words it counts each individual’s direct connections. The most relevant application can be found in Shih et al. (2010), who - while not putting it in SNA terms - claim that this measures the number of clients the incumbent and the most powerful competitor have in the Chinese Central Committee, and therefore their respective strength.\(^2\)

Betweenness has most famously been used by Padgett and Ansell (1993) to explain the rise of the Medici family in Renaissance Italy. It looks at all possible pairs of individuals, determines which is the shortest connecting path along network ties, and counts on how many such shortest paths each individual sits. Influence here does not depend on having direct authority over or easy access to others, but on “being in the right place”. An individual with high betweenness might be the only link between two separate groups, and thus have the possibility of manipulating or withholding information from others or extracting a brokerage premium.\(^3\)

Closeness or information centrality has so far only been used outside the context of authoritarian elites, as a measure of influence when spreading or receiving information through network ties. It is technically defined as the inverse of the sum of distances to all other individuals on the closest possible path. The measure thus ranges from 1 (a position that is connected with every other position in the network) to 0 (assigned to individuals without any connections, which is usually treated as being at an infinite distance from all

\(^2\)Mathematically, degree centrality of a node \(p_k\) is defined as \(C_D(p_k) = \sum_{i=1}^{n} a(p_i, p_k)\) where \(n\) is the number of nodes and \(a(p_i, p_k) = 1\) if and only if \(p_i\) and \(p_k\) are connected, 0 otherwise. (Freeman, 1979).

\(^3\)Betweenness centrality of a node \(p_k\) is defined as \(C_B(p_k) = \sum_{i=1}^{n} \sum_{j<i}^{n} \frac{g_{ij}(p_k)}{g_{ij}}\), where \(g_{ij}\) is the number of shortest paths linking two other nodes \(p_i\) and \(p_j\), and \(g_{ij}(p_k)\) is the number of such paths containing \(p_k\) (Freeman, 1979).
In the light of how individuals form coalitions in the model (see next section), closeness also seems very suitable to capture their ability to do this.

There are a number of more complex centrality measures that could also be relevant in this context. *Eigenvector centrality*, with its ability to capture “being connected to well-connected alters” certainly seems very promising, as well as the closely related *Bonacich centrality*, which also allows for a negative effect of being connected to alters with a large degree *Bonacich* (1987). Unfortunately, the program I used for the simulation (netlogo and the network extension) turned out to calculate the former incorrectly. Luckily, the error seems to have been fixed in the meantime. Future versions of this paper will therefore explore these and related measures in more detail.

### 3 The basic model with one contender

I start out with a very basic model of 50 elites, from which one tries to form a majority coalition. One might ask why I require the winning coalition to be a majority coalition, as the selectorate theory predicts smaller coalition sizes for authoritarian regimes. The majority condition assumes that each individual commands the same amount of power resources, an assumption I plan to relax in a future extension of the model (see footnote 5). There I will also show that even with equal distribution of those (external) power resources, the network structure (endogenously) reduces the necessary winning coalition size.

In this first model, the other 49 elites are absolutely identical, except for their position in the social network. The size of the network (50) is admittedly chosen more or less

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4 The closeness centrality of node $p_k$ is defined as: $C_c(p_k) = \sum_{i=1}^{n} \frac{1}{d(p_i, p_k)}$, where $d(p_i, p_k)$ is the number of ties on the shortest path linking $p_i$ to $p_k$ (*Freeman, 1979*). This measure is undefined if the network has more than one component, as some nodes are not reachable in that case. This is often resolved by computing closeness centrality separately for each component: see figure 6. This is not ideal for our purpose, as we would obviously expect individuals on small, disconnected components to be disadvantaged when it comes to forming a majority coalition, and therefore should actually have one of the lowest, not the highest score on that measure. I have corrected most of this mismeasurement by setting the closeness centrality of individuals not located on the largest component to 0.

5 An extension in which the elites have specific personal characteristics that make them more or less desirable coalition partners - e.g. their ideology on a left-right axis, or their possession of certain power resources or skills - seems an interesting next step. Future versions of this paper will certainly explore this at least as a robustness check and in order to establish the relative importance of the network structure compared to other factors.
arbitrarily, and future versions of this paper will explore how the overall size of the network influence the findings. However, there is no obvious reason why any of the findings presented here should not apply to networks of different size. But while the findings should apply to very large networks, the model’s assumptions may not: individuals might prefer to be in a coalition of a friend of a friend (and even his or her friend), but might be indifferent between a complete stranger (i.e. someone with whom they have not even an indirect connection) and a fourth or fifth degree friend.

The leader first tries to recruit anyone connected to him or her from among the 49 elites. If this does not result in a majority coalition, he or she will ask anyone connected to current coalition members to join, and so forth. Once the coalition reaches a size of 26 or more, the elites can be divided into at least 3 groups: the leader, individuals who are clearly not part of the coalition, and individuals who clearly are part of the coalition because they are “closer” to the leader than any other alternative member of the elite. There might be a fourth group: putative coalition members equidistant to the leader (between whom he or she is therefore indifferent) which are further away than those who have joined so far, but which are not all necessary to push the coalition from a minority to a majority. I will call this fourth group “expendables”.

The analysis of the first model simply summarizes the coalition that each of the 50 elites would create if they were to follow this rule as the leader. I vary the density of the network, by randomly deleting ties from a complete network (i.e. a network with all possible ties existing) until it becomes empty (i.e. with no existing ties). Finally, I also compare the results on such an Erdős-Rényi random network with those on a preferential-attachment network, a network that is more likely to have the typical structure of a social network, in particular that of a complex patronage network. For an example of how an Erdős-Rényi random network and a preferential-attachment one look like, see figure 6.

Note that the complete network is simply the basic versions of the selectorate theory, where there is no meaningful social structure imposed on the selectorate. In the complete network, all positions are equivalent: as in the model without social network, the leader immediately considers all other 49 elites for inclusion in the coalition. All of them are

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6The density of a network is measured as the number of ties present divided by the number of ties possible.
individually expendable, but at least 25 have to be chosen to form a majority coalition using some unspecified criteria not included in the model. As the individuals do not have any individual characteristics, the leader would have to chose at random.

Once some ties are removed, however, the network structure gains traction: some individuals are now randomly assigned to be closer to the leader (measured in the number of “hops” it takes them to reach him or her) than others - resembling the “affinity” parameter that Bueno de Mesquita et al. (2003) use to break ties.

4 Results from the basic model with one leader

4.1 How networks create hierarchies

At first one might assume that the results of such a random network should therefore be identical to the those of a model without network. But as I will show in this section, this is only the case for the complete network (or, more generally, for all regular or lattice networks).

Let’s look at one metric of interest, the number of coalitions in which a specific individual will be part, and the distribution of that metric in the population. Are there individuals that end up in more coalitions than others, i.e. are preferred coalition partners to many others? How common are they and how large is the difference between them and the average individual?

The complete network does indeed produce the same result as the selectorate model without network: each individual is the leader once and an expendable in every other coalition. If each leader chooses the 25 coalition members at random from the 49 expendables, the metric will have a binomial distribution centered at 26 (or at 52%): in every of the other 49 coalition building exercises, an individual has a 50% chance of being included (see figure 1 - the kernel density plots can be thought of a smoothed histograms).

However, that is only true for a very dense (Erdös-Rényi) random network, as the kernel density plots in figure 2 show. As the network becomes sparser, a small group of people emerges with a good chance of being included in almost any coalition that will be formed. On the left side of the plots, an increasingly large group of people is excluded
Figure 1: Kernel density lines for the distribution of the chance of ending up in the winning coalition. Each grey line represents one simulation, in which each of the 50 individuals were chosen as leaders who initiate a coalition once. The black line denotes the average over 100 such simulations. The distributions in a model without network (above) and in a complete network (below) are identical: in both cases, individuals most commonly are part of 52% (or 26) out of the 50 coalitions. Individuals that are part of less than 40% or more than 60% of the coalitions are relatively rare, and there are no individuals that never or always join a coalition.
from almost any coalition except the one they initiate themselves. The asymmetry of the distribution is caused by isolates (individuals without any ties) or individuals that form part of a smaller, disconnected component of the network, who cannot be reached by most coalition initiators.

One might argue that an Erdös-Rényi random network is unlikely to be an accurate representation of a network among the authoritarian elites, as it assumes random tie formation. Instead such elites are thought to form “(complex) patronage networks”. From the verbal description (and illustrations of Scott (1972) and Nathan (1973)) of those networks - patrons cultivating individual ties to clients, which are discouraged from forming ties among themselves, but might act as patrons for clients on their own - one would suspect that those networks resemble a structure also called complex star or tree network by network analysts. These networks are another workhorse of the network literature. Such a preferential-attachment network (Albert et al., 2000) involving 50 individuals would have roughly a density of 0.04.

As figure 3 shows, the results are very similar as in a random network of similar density: a very flat distribution in which a considerable proportion of the individuals are included in most of the coalitions, with another group rarely being invited to join. The differences at the right and left ends of the distribution can be explained by the fact that the preferential attachment network does not have disconnected individuals or components. Therefore, even the most peripheral individuals are included in some coalitions. On the other end, well-placed individuals are included in almost every possible coalition, because no coalitions are initiated by disconnected individuals or on separate components.

What insights can be gained from those simulations? Mainly that networks and connections create a stronger hierarchy the rarer they are: if almost everyone is connected to everyone else, then the outcome looks very similar to that without a network. But the sparser the network, the more unequal the positions of the individuals in the network: some are systematically left out, while others are almost guaranteed a place in the majority coalition, irrespective of who the initiator is.7

7There is an alternative explanation, namely that the hierarchy becomes stronger the further away we move from a regular network, one in which all individuals have the same number of connections (of which the complete network is one). Unfortunately, in this particular setup it is not possible to distinguish
Figure 2: Kernel density lines for the distribution of the chance of ending up in the winning coalition, for Erdős-Rényi random networks of different density. Grey lines represent one simulation, the black line the average over the 100 simulated networks, and the red line the baseline distribution of the model without network.
Figure 3: Kernel density lines for the distribution of the chance of ending up in the winning coalition, for Erdős-Rényi random networks and a preferential-attachment network of similar density. Grey lines represent one simulation, the black line the average over the 100 simulated networks, and the red line the baseline distribution of the model without network.
In more dense networks, everyone ends up with a roughly equal chance of being included in a specific coalition. However, even then it is not the case that falling above or below the mean is a matter of pure luck, as in the case of the model without network. If we know the network position of the individual, we can quite accurately predict his or her chances, as the next section will show.

4.2 Predicting success

The term “random” network in combination with a distribution that looks binomial should not mask the fact that an individual’s position in the hierarchy is in fact predictable. In this section, I will illustrate how one’s chance of ending up in a (majority) coalition is strongly correlated with one’s centrality in the network.

In order to test the predictive power of those three measures, I conduct two tests on each network constructed (i.e. each run of the simulation). Firstly, a simple correlation between each of the measures with the outcome of interest, which is the number of majority coalitions an individual is able to join. Secondly, a regression using all three measures as independent variable. Centrality measures are usually at least moderately correlated, so it is worth disentangling the effect of the three different aspects of informal power and influence.

Figure 4 displays the result of the first test by summarizing the three correlation coefficients of the hundred networks constructed for each level of density using kernel density lines. The colors of the lines follow the order of the rainbow in ascending order, from red (0.02) to purple and pink (0.5).

The number of individuals one is connected to (degree) is highly correlated with the percentage of majority coalitions that one may join: the correlation coefficients vary mostly between moderate (0.5) and very strong (0.95). But closeness centrality is even more strongly correlated: except for networks with density between 0.2 and 0.25, the correlation is higher than 0.8. The small “bumps” in the region between 0.2 and 0.6 are likely due to the way netlogo treats small, separate components as mentioned in footnote 4 earlier. The correlation with betweenness centrality is a bit weaker, but still at least

\[ \text{between those two explanations.} \]
Figure 4: Kernel density lines on the strength of correlation between different centrality measures and the probability of ending up in a majority coalition in 100 E-R networks constructed for each level of density. The colors of the lines indicate the density and follow the order of the rainbow in ascending order, from red (0.02) to purple and pink (0.5).
Looking at networks of different densities, a general pattern emerges as well, with coalition formation in networks of intermediate density (0.2 and 0.25, blue and purple) tending to be less easy to predict as indicated by the weaker correlation coefficients. This is also reflected in their much lower AIC in the regression models below.

Figure 5 displays the distribution of the size of the coefficient on the centrality measures in a regression model in a similar manner. Note that the absolute size is of course dependent on the unit of analysis: betweenness centrality is not standardized and could in theory range up to 2500, while closeness centrality is bound by 1, and degree limited at 49 (none of them can be negative). A one unit change in closeness centrality is thus quite different from one in betweenness centrality.

The results are consistent across all but the most (0.5 - pink) and least (0.02 - red) dense networks under consideration: once closeness centrality is included, both degree and betweenness centrality largely lose their significance in predicting the chance of joining a majority coalition. The latter two’s mean regression coefficient in the 100 simulations for each density is close to zero, with a more or less equal distribution both above and below zero.

That on closeness centrality, however, remains above zero in about 90-95% of all simulations, except for the extreme densities. In the network with the lowest (red - 0.02) density, the most important indicator likely is whether the individual is a part of the main component or not, and not his or her position within that component. The other exception is the most dense network (pink - 0.5), where the closeness coefficient varies widely between different simulations around 0, while the coefficient on degree is positive in a large part of the simulations. In such a network where half the possible ties are present, the average individual has just enough connections to establish a majority coalition with those directly connected to him or her. Therefore his or her indirect connections (captured in the closeness centrality measure) are not relevant.\textsuperscript{8}

What are the broader conclusions for the analysis of informal connections in authoritarian coalition formation? One thing to notice is that the direct connections, which are

\textsuperscript{8}The same is partly true for the 0.25 density network (purple) as well. The “bump” with the 0.04 network (orange) is presumably due to the issues discussed in footnote 4.
Figure 5: Kernel density lines on the size of the coefficient of different centrality measures on the probability of ending up in a majority coalition in 100 E-R networks constructed for each level of density. The colors of the lines indicate the density and follow the order of the rainbow in ascending order, from red (0.02) to purple and pink (0.5).
the only ones that researchers of authoritarian regimes have examined so far, are actually not the most important ones, or at least not the only relevant ones. Looking at the whole network instead of only the network neighborhood of a few select individuals helps us understand better who will end up in a majority coalition.

It also points to a possible problem with the analysis that only considers ties to patrons, as Shih et al. (2012); Jia et al. (2013); Zhang (2009), and Choi (2012) do. As a higher number of connections will mechanically increase the chance that one is connected to a patron, candidates who are supposed to have profited from their special connections to a patron might simply have profited from being well-connected overall. This would not invalidate the general finding that connections matter, but it would change the story about the mechanisms through which elite networks exert their influence.

5 The model with competitor

So far, the model has given little agency to most of the individuals involved: anyone in the network automatically joined the coalition if he or she received an offer to do so, presumably because being in the coalition was more desirable than being left out. But why wouldn’t one of the left-outs start their own coalition, something akin to organizing a revolution? Or why shouldn’t a coalition member decide that forming his or her own opposition coalition is more attractive, the equivalent of initiating a palace coup?

The second model examined here thus contains two leaders: an incumbent and a competitor who try to form their favorite coalition simultaneously. The other elites now have the choice of joining either one of them. Like the leaders, they prefer individuals closer to them and will therefore join the leader who is fewer “hops” away.

Once the leaders have decided who they want to include and the followers who they want to join, we can let the sizes of the coalitions determine the winners. Figure 6 gives an example for competing coalition formation in an Erdös-Rényi random network (right) and a patronage / preferential-attachment network (left).

Except if this headcount results in a draw, each individual will have one the following statuses: they can either be the leaders of the majority or the minority coalition (winner or loser - red or green in figure 6); they can be members of the losing or the winning coalition
(pale green or pink), or they can be unaligned (grey). The latter two actually have important subcategories, because as in the first model, there may again be disposables in the winning coalition, individuals that are not all needed to reach the necessary size for a majority coalition, and which are all equidistant from the leader. The situation among the unaligned is even more complicated: those that are equidistant to both leaders have no a priori preference between them. If they are close enough to both, they might be kingmakers (individuals 7, 9, 10, 11, 14, 17, and 44 in the E-R network in figure 6), especially in the case of a draw. I will call those “waverers”. But there might also be “left-outs”, individuals that are so peripheral that the winner can form a majority coalition without them, and at the same time so cut off from the loser that the latter cannot mobilize them for his side either (the grey individuals in the preferential-attachment network in figure 6).

However, while it is again interesting to analyze why some individuals are more likely to end up in one of those categories over the other, the following will first focus on a question that is an extension of what is discussed in the previous section: can we go beyond identifying which individuals are more likely to be in the final coalition to predicting which coalition will emerge? In other words, are there networks in which one leader cannot be removed by any (single) opponent’s coalition? In real life, this stable coalition could be arrived at through trial and error: several candidates propose coalitions or attack the existing coalition one after the other until the right leader emerges. Or it could emerge right away, if all individuals know the network structure and refrain from initiating coalitions that will be beaten anyway. As it turns out, networks with a unique, dominating coalition are not that uncommon.

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9There is in fact another type of waverer, whose ability to influence the outcome is more conditional: this position is equidistant to both competitors, but the only way for any of them to reach that waverer is by going through another waverer. The other waverer might thus determine the affiliation of a number of individuals to which he or she holds sole access. Examples would be 9 in the E-R network, which is blocked by 44, and 10 and 11, blocked by 7.
Figure 6: Three steps in the coalition formation process on a preferential-attachment (left) and Erdős-Rényi network (right) with similar density (0.04). In the middle row, both contenders have started recruiting their direct connections into their coalition (colored in light green and pink, respectively). The bottom row presents the final headcount: red wins 23:15 in the preferential-attachment network, green 18:14 in the E-R network.
Figure 7: Left: probability of a the competing coalitions being of equal size (ignoring waverers). Right: probability of draw or that one or several waverers could turn minority into majority coalition.

6 Results of the model with two competitors

6.1 Instability of dense networks: dominating coalitions are possible, but dependent on waverers

The first thing to notice is that the outcome in a complete network (density = 1), an empty network (density = 0), and in the situation without network is quite similar: the leaders of both coalitions will be the only members of their coalition, thus leading to a draw. In the case of the empty network, everyone else is a left-out, in the other two cases they are all waverers. The winning coalition would thus be determined by the rule according to which waverers pick their side. But a draw really only has a large chance of occurring in a very dense or very sparse networks, as the left plot in figure 7 shows.

But the strategic decisions of waverers can have an influence not just in the case of a draw: if the difference between the sizes of the two coalitions is small, then they could turn a minority into a majority. The plot to the right in figure 7 displays exactly that: the chance of a draw occurring or a situation in which waverers could at least provoke a draw if they all decided to support the minority coalition. In sparse networks, both plots look quite similar, but in denser networks, the leader of the larger coalition will almost always have to convince the waverers to at least abstain.

This difference is almost completely due to the higher number of waverers present: density and the number of waverers are highly correlated (0.92). The more ties each
individual in the network has, the higher the chances that he or she is directly connected to both competitors.

The analysis conducted so far does not tell us anything about whether we will have an undisputed coalition emerging in a network, however. After all, a leader might have a clear majority against one opponent, but easily lose against another one. Ideally a leader would want to “dominate”: be able to form a majority coalition against each and every possible opponent emerging from that network. In particular, we would like to avoid the classical coalition formation problem of cycles: a situation in which leader B’s coalition beats leader A’s coalition, only to get beaten by a third coalition (C), which in turn can be beaten by A, etc.

The question of interest thus becomes: are there individuals who dominate the network so much that their coalition can beat any other leader’s coalition? And going one step further: are there maybe even leaders so overwhelmingly dominating that they can win even if the opposition manages to recruit all waverers?

The left part of figure 8 shows that first condition is actually quite common. Between about two thirds and four fifths of all simulated Erdős-Rényi random networks of different density had a unique winner. Even among many of the remaining networks there is a near winner: in about half of them, there is one leader who wins against 48 of the 49 competitors and manages a draw against the remaining leader.

The picture changes if we demand that the leader’s coalition should win overwhelm-
ingly against each possible opponent, that is even when the waverers join the opposition. These sort of dominating leaders are indeed quite rare, except in relatively sparse networks, as the plot to the right in figure 8 shows. In dense networks, the leaders thus at least have to prevent the opponent from recruiting (the much more common) waverers. The latter thus become kingmakers who might switch allegiance if they feel that the coalition does not benefit them sufficiently.

Such potentially destabilizing situations are more common in dense networks, which might appear counter-intuitive: shouldn’t a dense trust network lead to a more stable rule? But a dense network also means that many individuals hold conflicting loyalties. In the face of an outside threat, dense elite ties can prevent a split. But in internal conflicts, the large group of waverers with dubious loyalty could lead to constantly shifting alliances without a clear leadership emerging.

6.2 Dominating coalitions in different networks: the stability of patronage networks

Interestingly, there is initially very little difference between the results of the patronage (i.e. preferential-attachment) network and an Erdős-Rényi network of similar density: the probability of a draw occurring is 0.034, that of draw or waverers overturning or neutralizing a majority 0.035 - as opposed to 0.033 and 0.034 in the E-R network. The assessment changes if we look at how wins are distributed among the network positions: not only is there one leader that can beat all other opponents in almost every (i.e. 96% of all) preferential-attachment network simulated. The number remains equally high even if we demand that his or her coalition should be able to beat the opposition and the waverers combined. In an Erdős-Rényi network, the corresponding numbers is much lower: 71% and 48%, respectively (see also figure 8).

Figure 9 shows the distribution of each leader’s coalition strength in 100 such network simulations, measured in how many of the 49 possible opposition coalitions they can beat with their coalition. The distribution is relatively uniform in all networks, with the exception of a large spike of perpetual losers located at the edge of the network or detached from it in the E-R networks. The spike of leaders who are unable to beat even
one opponent on the leftmost side of the histogram is less pronounced in preferential attachment networks because they, by construction, do not have any of the detached isolates.

Leaders who can beat all competitors (the rightmost bar at 49 in the histograms in figure 9) seem to be rarer in E-R networks, especially if they need to secure a victory despite the waverers joining the opposition (bottom right plot). However, further simulations are probably needed to explore if this difference is indeed significant and what exactly causes it. One important observation is certainly that while situations with no waverers are roughly equally common, there can be at most one waverer in a preferential attachment network. E-R random usually have several.

Nevertheless, the results do not seem to support the claim - commonly found in the literature - that factions or patronage networks lead to instability through infighting or “bandwagoning” (Tsou, 1995). The emergence of a dominating leader is, if anything, more likely in the preferential-attachment network resembling a patronage network than in a E-R random network. And waverers, whose unclear loyalty could cause a sudden mass rush to a (new) majority coalition, even seem to be less influential and certainly
6.3 Who are the dominating leaders and their followers?

What position do those dominating leaders hold? Unsurprisingly, centrality matters again: average degree, closeness, and betweenness of dominating leaders are significantly higher than those of other individuals at all levels of density. If all three measures are used to predict whether someone will become a dominating leader, degree again becomes insignificant at network densities above 0.06. However, this time betweenness centrality is the most robust predictor of whether an individual is able to form a dominating coalition. The result holds for both random and patronage networks, and also if only “overwhelming dominators” are taken into account.

Betweenness, as mentioned above, indicates a strategic positions between relevant parts of the network, and is measured by the number of shortest paths between two individuals passing through the position. A leader in that position should therefore be able to block many opponents from reaching a large number of potential followers quickly. But at most one individual can be a dominating leader in each of the 100 simulations per network conducted so far - more simulations will be needed to explore this basic finding and provide explanatory graphics.

7 Conclusions

In this paper, I have constructed an simple agent-based model of coalition formation along network ties, and explored it on two different network types and networks of different density. I have found that there are hierarchies in terms of the likelihood of ending up in the winning coalition and in terms of being a stable leader in all but the regular (i.e. complete Erdős-Rényi random) networks. The leaders of those hierarchies can be identified through network centrality measures. Individuals with high closeness centrality are popular coalition partners and therefore more likely to be included in the winning coalition.
coalition. Coalition leaders with high betweenness centrality are most likely to fend off all possible competitors, thereby maintaining a stable rule.

What are the broader implications of the model for the analysis of informal connections in authoritarian coalition formation? The analysis of the model with one competitor has shown, for instance, that degree centrality - an individual’s number of connections - can predict the likelihood of joining the inner circle of an authoritarian regime. Shih et al. (2012); Jia et al. (2013); Zhang (2009), and Choi (2012)’s findings that individuals with patrons are more likely to rise in the party hierarchy could thus have an alternative explanation: maybe the promoted individuals were simply more connected to anyone, including the alleged patrons. If this were the case, then we would still conclude that connections matter - but the main driver would not be the vertical connections to the center, but the horizontal connections to peers. In other words, it would be a story about a limited form of accountability to a wider party base instead of a centralized appointment procedure directed by a handful of leaders.11

In order to differentiate those two hypotheses, we need to measure the whole network. This is important also for other reasons: while direct connections are important, their importance may lie in the access they grant to indirect connections. As the analysis of the model’s results has shown, measures of direct connections (degree) are not significantly correlated with the chance of ending up in the winning coalition or being a stable leader once more complex measures (closeness and betweenness centrality, respectively) have been accounted for.

The model thus makes a powerful case for the use of social network analysis when examining the formation of winning coalitions and appointments to high-level positions in authoritarian regimes. With centrality measures, social network analysis might even hold the key for identifying grey eminences, patrons, and other holders of informal power in a more rigorous manner, and shed light on the source of power of someone without an official position granting command over resources and people. The model proposes as source their unique network position, which makes it difficult to assemble a coalition to successfully challenge them.

11In my empirical research on elites in the Chinese Communist Party, I do indeed find that while connections to patrons matter, so does the number of connections to other elites.
It also provides a series of other insights that should be explored more carefully: dense elite networks do not necessarily provide more stable regimes, and patronage networks appear to provide clearer and more stable leadership and ruling coalitions - as long as no individuals are removed. The example illustrated in figure 6 also seems to indicate that in a selectorate structured by a social network, the winning coalition does not necessarily need to be a majority.

Unfortunately, the program I have used for the simulations has some limitations, and the more complex extensions discussed in the paper (including alternative selection criteria, exploring different centrality measures, or the minimal size needed for a winning coalition) might have to be executed in R or Python.
References


