Unifying Configurational Comparative Methodology: Generalized-Set Qualitative Comparative Analysis

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ABSTRACT

Crisp-set (csQCA), fuzzy-set (fsQCA) and multi-value Qualitative Comparative Analysis (mvQCA) have emerged as distinct variants of QCA, with the latter still being regarded as a technique of doubtful set-theoretic status. Textbooks on configurational comparative methodology have emphasized differences rather than commonalities between these three variants. This article has two consecutive objectives, both of which focus on commonalities. First, but secondary in importance, it demonstrates that all set types associated with each QCA variant can be combined within the same analysis by introducing a standardized notational system. By implication, any doubts about the set-theoretic status of mvQCA vis-à-vis its two sister variants are removed. Second, but primary in importance and dependent on the first objective, this article introduces the concept of the multivalent fuzzy set. This set type forms the basis of generalized-set Qualitative Comparative Analysis (gsQCA), an approach that integrates the features peculiar to mvQCA and fsQCA into a single framework while retaining routine truth table construction and Boolean minimization procedures. Under the concept of the multivalent fuzzy set, all existing QCA variants become special cases of gsQCA.

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Qualitative Comparative Analysis (QCA) has made some important contributions to the toolbox of the social sciences in general, and the inventory of configurational comparative methods in particular.[1] Suitable for analyzing set-theoretic hypotheses, its diffusion across many disciplines of the social sciences has been one of the most notable methodological developments in recent years (Thiem and Duşa, 2013*c*:1-3).

For about a decade, QCA has also been diversifying into three separate variants. Textbooks now differentiate between crisp-set QCA (csQCA) (Rihoux and De Meur, 2009), multi-value QCA (mvQCA) (Cronqvist and Berg-Schlosser, 2009) and fuzzy-set QCA (fsQCA) (Ragin, 2009). Each of these variants is associated with a distinctive set type and has partly required different software with tailored routines (Thiem and Duşa, 2013*a*). For example, fs/QCA (Ragin and Davey, 2009) has long been the only program for fsQCA, and mvQCA could only be performed by Tosmana (Cronqvist, 2011). More recently, the fuzzy package for Stata (Longest and Vaisey, 2008) and Kirq (Reichert and Rubinson, 2013) have been introduced as alternatives to fs/QCA. In addition, the two R packages QCA (Thiem and Duşa, 2013*b*) and QCA3 (Huang, 2012) now offer extensive functionality for all three variants. Although software capabilities still differ considerably, end-users have almost always been able to carry out all the steps that their analyses have necessitated.

In spite of numerous methodological innovations over the last decade, however, one problem remains unaddressed. If the data do not fit certain combinations of set types associated with any of its three variants, QCA can either not be employed at all or analysts are forced to accept a loss of information by recalibrating sets into a processable format. While set types for mvQCA and csQCA can be reconciled as seamlessly as those for fsQCA and csQCA, multi-value and fuzzy sets have been incompatible so far. The dissolution of this discrepancy, albeit significant on its own, is the secondary objective of this article.

Its primary objective takes the argument one step further, but it heavily rests on the prior dissolution of the divide between multi-value and fuzzy sets. Ultimately, it is to be demonstrated that all three established QCA variants represent special cases of a more comprehensive variant, which shall be referred to as generalized-set Qualitative Comparative Analysis (gsQCA). At the core of gsQCA lies the multivalent fuzzy set, a set type that integrates the defining features of multi-value and fuzzy sets, while leaving established truth table construction and Boolean minimization procedures fully in place. In this connection, it is thus also to be argued that mvQCA should neither be regarded as a medium-n compromise between csQCA and fsQCA (Herrmann and Cronqvist, 2009), nor as a variant of doubtful set-theoretic status as some methodologists have recently argued (Schneider and Wagemann, 2012:258-263; Vink and van Vliet, 2009).

The article is structured around three parts. The first sections reviews the debate on the commonalities and differences between csQCA, mvQCA and fsQCA. At the same time, fundamental key concepts of QCA are defined in order to provide a consistent terminological basis for the remainder of the argument. Subsequently, the second section introduces an approach to combining crisp, multi-value and fuzzy sets in a single analysis. And finally, the third section generalizes the method proposed in the preceding section by introducing gsQCA. This consecutive structure follows a logic of increasing abstraction. The combination of all existing set types in a single analysis is novel but still adheres to some rules under which current QCA variants operate, such as the Min-Max theorem.[2] These will not hold true any more in gsQCA. The conclusions recapitulate the argument.

The State of the Debate

Charles Ragin's landmark work "The Comparative Method" (1987) introduced csQCA as the first of the three variants to a wider audience of social scientists.[3] Until the publication of Ragin (2000), csQCA was still called "QCA" because no ambiguities as to the acronym's meaning had existed. With twenty applications in published journal articles in 2011, it remains the most popular variant (Thiem and Duşa, 2013*c*:2).

The basis of csQCA is Boolean algebra, a theory of links between mathematical structures that differs in a number of aspects from the linear algebra of vector spaces, which the vast majority of social scientists are more familiar with. Boolean algebra has long been used in many areas of the natural and technical sciences, including genetic biology (Thomas, 1973), information technology (Brayton et al., 1984) and electrical engineering (Edwards, 1973), but it was the latter in particular from which QCA has borrowed many of its concepts. Special cases of Boolean algebra include propositional logic, switching-circuit theory and the theory of sets, but any set $\mathcal{S}$ of elements $\{\mathbf{S}_1\left\langle\mathbf{S}_1, \mathbf{s}_1\right\rangle, \mathbf{S}_2\left\langle\mathbf{S}_2, \mathbf{s}_2\right\rangle, \mathbf{S}_3\left\langle\mathbf{S}_3, \mathbf{s}_3\right\rangle, \ldots\}$ and two binary operations "+" (Boolean sum) and "·" (Boolean product) form a Boolean algebra if, and only if, for any $a \neq b \neq c$ and j, the following four axioms (A_1) to (A_4) hold: first, the operations are commutative:

$$\mathbf{S}_a + \mathbf{S}_b = \mathbf{S}_b + \mathbf{S}_a \text{ and } \mathbf{S}_a \cdot \mathbf{S}_b = \mathbf{S}_b \cdot \mathbf{S}_a; \qquad (A_1)$$

second, each of the operations distributes over the other:

$$\mathbf{S}_a \cdot (\mathbf{S}_b + \mathbf{S}_c) = \mathbf{S}_a \cdot \mathbf{S}_b + \mathbf{S}_a \cdot \mathbf{S}_c \text{ and } \mathbf{S}_a + \mathbf{S}_b \cdot \mathbf{S}_c = (\mathbf{S}_a + \mathbf{S}_b) \cdot (\mathbf{S}_a + \mathbf{S}_c); \qquad (A_2)$$

third, identity elements exist:

$$0 + \mathbf{S}_j = \mathbf{S}_j \text{ and } 1 \cdot \mathbf{S}_j = \mathbf{S}_j; \qquad (A_3)$$

and fourth, for each $\mathbf{S}_j \in \mathcal{S}$ there exists an $\mathbf{s}_j \in \mathcal{S}$ such that:

$$\mathbf{S}_j + \mathbf{s}_j = 1 \text{ and } \mathbf{S}_j \cdot \mathbf{s}_j = 0. \qquad (A_4)$$

Both $\mathbf{S}_j$ and $\mathbf{s}_j$ make up the set of exhaustive, disjunct *values* (categories) of the Boolean variable $\mathbf{S}_j$. Since classical Boolean variables are always dichotomous, the variable itself usually carries the label of one of its values. For example, if the Boolean variable $\mathbf{S}_j$ is labeled "social market economy" in a study on the economic systems of post-World-War II European states, then all objects that fulfill the requirements of the study's definition of a social market economy are assigned to the first of its values, $\mathbf{S}_j$. All other objects are assigned to the second value, $\mathbf{s}_j$. No object can be simultaneously assigned to both values.

Notably, Boolean-algebraic distributivity is false in linear algebra. Axiom (A_4) relates to a defining characteristic of crisp sets in that objects can only have either full membership in the set formed by one of a Boolean variable's two values or none at all. Expressed in the set-theoretic nomenclature of QCA, the union $\mathbf{S}_1 \cup \mathbf{s}_1$ of $\mathbf{S}_1$ and its complement $\mathbf{s}_1$ yield the universal set $\mathbf{U}$, their intersection $\mathbf{S}_1 \cap \mathbf{s}_1$ the empty set $\varnothing$.[4]

The ultimate goal of QCA is the reduction of a complex system of relations between a number of input variables and a number of output variables to simpler equivalents by eliminating redundant inputs. Input variables are referred to in QCA as *condition sets*, output variables as *outcome sets*. The process of input variable elimination is generally called *Boolean minimization*.[5] The system of relations between input variables and output variables represents a Boolean function.

From axioms (A_1) to (A_4), a number of theorems can be derived, two of which are of especial importance for minimizing Boolean functions. The first leads to the elimination of as many condition sets as possible from intersections of condition sets and the second to the elimination of as many intersections as possible from a union of intersections. Formally, for any sets $\mathbf{S}_a$, $\mathbf{S}_b$ and $\mathbf{S}_c$, $\mathbf{S}_a\mathbf{S}_b \cup \mathbf{s}_a\mathbf{S}_b = \mathbf{S}_b$ and $\mathbf{S}_a\mathbf{S}_b\mathbf{s}_a \cap \mathbf{S}_c\mathbf{S}_b\mathbf{S}_c = \mathbf{S}_a\mathbf{S}_b \cup \mathbf{s}_a\mathbf{S}_c$.[6]

If $\mathbf{U}$ represents the universal set of objects with any subset denoted by $\mathbf{S}_j$, $\mathbf{S}_j$ can itself be represented by its *characteristic function* $\mu_{\mathbf{S}_j}$ as given in Equation (1). It assigns a value of "1" or "0" to all subsets of $\mathbf{U}$ which share or do not share the property denoted by $\mathbf{S}_j$ (cf. Klir, St. Clair and Yuan, 1997:63).

$$\mu_{\mathbf{S}_j}(u) = \begin{cases} 1 & \text{if } u \in \mathbf{S}_j \\ 0 & \text{if } u \notin \mathbf{S}_j \end{cases} \tag{1}$$

If the Boolean-algebraic system is restricted to the two indicators "0" and "1", these numbers may not only represent symbols of membership and non-membership in any value of $\mathbf{S}_j$, but also numerical indicators of the degree to which some object $u_i \in \mathbf{U}$ belongs to $\mathbf{S}_j$ or not. Mapping objects into the binary set of *membership scores* $\mathbb{B} = \{0, 1\}$ by means of characteristic functions therefore allows the representation of superset and subset relations as functional inequalities of the form $\mu_{\mathbf{S}_a}(u) \leq \mu_{\mathbf{S}_b}(u)$.

With this convention in place, operations on sets can also be written as operations on their characteristic function. For example, the characteristic function of the union of sets $\mathbf{S}_a$ and $\mathbf{S}_b$ can be expressed by means of the parallel maximum of the individual characteristic functions as given in Equation (2).

$$\mu_{\mathbf{S}_a \cup \mathbf{S}_b}(u) = \max[\mu_{\mathbf{S}_a}(u), \mu_{\mathbf{S}_b}(u)] \tag{2}$$

Analogously, the characteristic function of the intersection of $\mathbf{S}_a$ and $\mathbf{S}_b$ can be constructed using the parallel minimum of the individual characteristic functions as given in Equation (3).

$$\mu_{\mathbf{S}_a \cap \mathbf{S}_b}(u) = \min[\mu_{\mathbf{S}_a}(u), \mu_{\mathbf{S}_b}(u)] \tag{3}$$

If any exhaustive list of unique characteristic functions involving the intersection of all condition set values $\{\langle \mathbf{S}_a, \mathbf{s}_a \rangle, \langle \mathbf{S}_b, \mathbf{s}_b \rangle, \langle \mathbf{S}_c, \mathbf{s}_c \rangle, \ldots\} \in \mathcal{S}$ such that no value is included together with its complement is joined by an *outcome value* of some operation on any number of elements in $\mathcal{S}$, then this object is called a *truth table*, or table of combinations (cf. McCluskey, 1965:75). There exist $d = \prod_{j=1}^{k} p_j$ characteristic functions of p-valued k condition sets. These functions are more commonly referred to in QCA as *configurations*.

A hypothetical truth table with three condition sets $\mathbf{C}_1$, $\mathbf{C}_2$ and $\mathbf{C}_3$ and their corresponding outcome value OUT is presented in Table 1. Three condition sets with two values each yield eight configurations $\mathcal{C}_1, \mathcal{C}_2, \ldots, \mathcal{C}_8$. The minimum number of cases n that is usually required for the respective outcome value is given in addition.

TABLE 1 ABOUT HERE

A condition set is the analogue of an independent variable, but the outcome value is not the same as the dependent variable. The dependent variable is captured by the outcome set, which does not show up in QCA truth tables. Instead, the outcome value represents a truth value indicating whether the evidence is consistent with a hypothesis about the existence of a subset relation between each configuration and the outcome set or not. Configurations 1-4 are *positive* because they confirm the hypothesis (OUT = 1), configuration 5 is *negative* because it falsifies the hypothesis (OUT = 0). Mixed evidence exists for configuration 6 (OUT = C). If at least two objects conform to one configuration, but the evidence neither sufficiently confirms nor falsifies the hypothesis, *contradictions* arise. No empirical evidence exists for configurations 7 and 8 (OUT = ?). If a configuration contains no or too few objects, it is called a *logical remainder*. Remainder configurations are theoretically possible but unobserved combinations of condition set values.

The *canonical union* resulting from the truth table presented in Table 1 is given by set relation (S_1). It consists of four *fundamental intersections*, each of which corresponds to a positive configuration from Table 1.

$$\overbrace{\mathbf{C}_1\mathbf{C}_2\mathbf{C}_3}^{\mathcal{C}_1} \cup \overbrace{\mathbf{C}_1\mathbf{C}_2\mathbf{c}_3}^{\mathcal{C}_2} \cup \overbrace{\mathbf{C}_1\mathbf{c}_2\mathbf{C}_3}^{\mathcal{C}_3} \cup \overbrace{\mathbf{C}_1\mathbf{c}_2\mathbf{c}_3}^{\mathcal{C}_4} \subseteq \mathbf{O} \qquad (S_1)$$

If two such intersections differ on all values of one condition set only, then this set is redundant and can be eliminated so that a simpler intersection, called *implicant*, results. Figure 1 shows how (S_1) can be minimized in two passes. In the first pass, the four three-way fundamental intersections can be minimized to four two-way implicants. In the second and final pass, these four implicants can then be minimized at once to a single one-way *prime implicant*. No further reductions are possible. $\mathbf{C}_1$ is the only condition set value that is essential in relation to the specified outcome set value.[7]

FIGURE 1 ABOUT HERE

While the logic of crisp sets is fundamental, a considerable number of social-scientific concepts do not neatly fit into the binary structure of classical Boolean algebra. In order to address this shortcoming, Zadeh (1965) has proposed the notion of the fuzzy set, which is at the core of the fsQCA variant (Ragin, 2000).[8] The most important feature of fuzzy-set theory is the invalidity of the fourth Boolean-algebraic axiom (A_4). In contrast to membership in a crisp set, membership in a fuzzy set is not limited to the binary set $\mathbb{B} = \{0, 1\}$ but extends to the unit interval $\mathbb{U} = [0, 1]$. With fuzzy sets, it becomes possible for an object to have non-zero membership in $\mathbf{S}_j$ and $\mathbf{s}_j$ at the same time.[9] The distinction between crisp and fuzzy sets is, however, one of degree, and not one of kind. From the perspective of set-theory, crisp set membership simply represents the border case of fuzzy set membership (cf. Clark et al., 2008:57).

The fsQCA analogue of the characteristic function in csQCA is the *membership function*. As the only restriction on membership functions is a co-domain in the unit interval, they can take an infinite number of forms (Verkuilen, 2005). For example, the membership function suggested by Ragin (2008) and implemented in the fs/QCA software is based on a specific instance of the logistic function class.[10]

All variants of QCA require the construction of truth tables, but with fuzzy-set data, truth tables cannot be constructed directly because the possible number of configurations would be infinite. The truth table algorithm introduced in Ragin (2008) thus considers configurations as the outer corners of a k-dimensional vector space. Although each object can have membership in one, several or all dimensions of this space, it can only be a strong instance, that is, have membership above 0.5, in a single configuration. Usually, more objects than only those that represent strong instances have some membership in a configuration, so its outcome value depends on the partial configuration membership of all objects. Despite these particularities of truth table construction in fsQCA, the process of Boolean minimization remains exactly the same.

About four years after fsQCA had been introduced, Cronqvist (2004) presented the Tosmana software for processing multi-value sets (Cronqvist and Berg-Schlosser, 2009; Cronqvist, 2011). The first substantive application of mvQCA has come from Balthasar (2006), but only six further articles have been published since (Thiem, 2013). This imbalance may be explained by the fact that, so far, mvQCA has been viewed with a considerable degree of suspicion by methodologists (Schneider and Wagemann, 2012; Vink and van Vliet, 2009), which in turn may have deterred end-users from employing it.[11] Why this suspicion has been unjustified is explained in more detail elsewhere (Thiem, 2013).[1] It can be shown, however, that mvQCA represents a set-theoretic generalization of csQCA on the dimension of condition set values, while leaving membership crisp.[12] Multi-value sets simply cluster together more than two condition set

[1]See also the response by Vink and van Vliet (2013).

values such that $\mathcal{S}$ is extended to

$$\mathcal{S} = \{\mathbf{S}_1 \langle \mathbf{S}_{1,1}, \mathbf{S}_{1,2}, \ldots, \mathbf{S}_{1,l}, \ldots, \mathbf{S}_{1,p} \rangle, \mathbf{S}_2 \langle \mathbf{S}_{2,1}, \mathbf{S}_{2,2}, \ldots, \mathbf{S}_{2,l}, \ldots, \mathbf{S}_{2,p} \rangle, \\ \mathbf{S}_3 \langle \mathbf{S}_{3,1}, \mathbf{S}_{3,2}, \ldots, \mathbf{S}_{3,l}, \ldots, \mathbf{S}_{3,p} \rangle, \ldots\}.$$

Then, the negation of each value in each condition set is the Boolean sum of each element in the set of all other values in that condition set. Formally, since

$$\begin{aligned} \mathbf{S}_{j,l=\mathrm{z}} &= (1 - \mathbf{S}_{j,1}) \cdot (1 - \mathbf{S}_{j,2}) \cdot \ldots \cdot (1 - \mathbf{S}_{j,l\neq\mathrm{z}}) \cdot \ldots \cdot (1 - \mathbf{S}_{j,p}) \\ &= 1 - (\mathbf{S}_{j,1} + \mathbf{S}_{j,2} + \ldots + \mathbf{S}_{j,l\neq\mathrm{z}} + \ldots + \mathbf{S}_{j,p}) \end{aligned}$$

by De Morgan's Theorem, it follows that

$$1 - \mathbf{S}_{j,l=\mathrm{z}} = \mathbf{S}_{j,1} + \mathbf{S}_{j,2} + \ldots + \mathbf{S}_{j,l\neq\mathrm{z}} + \ldots + \mathbf{S}_{j,p}.$$

For this reason, multi-value sets never carry the label of one of its values, unlike traditional sets as used in csQCA, but all axioms and theorems of Boolean algebra remain valid in mvQCA.

After the introduction of fsQCA and mvQCA, the classical approach to configurational comparative thinking popularized by Ragin (1987) has come to be referred to as csQCA. The next section will demonstrate that, first, csQCA, mvQCA and fsQCA are more closely related to each other than textbooks have acknowledged and second, that mvQCA fits squarely into the set-theoretic framework of QCA despite recent claims to the contrary (Schneider and Wagemann, 2012; Vink and van Vliet, 2009). Notwithstanding its significance, this reconciliation represents only an intermediate objective, but one that is necessary for the introduction of gsQCA in the third section.

Combining Crisp, Multi-Value and Fuzzy Sets

This section lays out how crisp, multi-value and fuzzy sets can be combined in a single analysis. In order to harmonize their different notational systems, three definitions are given first. The set type that has traditionally been referred to as *crisp set* is relabeled to *bivalent crisp set*, *multi-value set* to *multivalent crisp set*, and *fuzzy set* to *bivalent fuzzy set*. The prefixes *bi-* and *multi-* do not refer to set value membership but the number of set values.[13] Even though an object's membership in them can be graded, fuzzy sets also only have two values.[14]

After the three basic set types have been defined, a common notational system can be established. The most serious point of criticism raised by Vink and van Vliet (2009) and echoed by Schneider and Wagemann (2012:258-263) has been that mvQCA is difficult to understand in set-theoretic terms. Such difficulties do not arise from mvQCA itself, but they are a consequence of the different systems of notation currently used in csQCA and fsQCA on the one side, and mvQCA on the other. Both csQCA and fsQCA have relied on *membership score notation*, mvQCA on *value notation*. Parallels have often been drawn between the numbers used in these two systems, although they are not comparable.

Membership score and value notation can be unified by putting them on a common denominator. The resulting system will be referred to as *value-score notation* (VSN). VSN borrows from mvQCA insofar as curly brackets indicate set values, but this indicator itself does not imply full membership in that value. Instead, VSN draws on csQCA and fsQCA by appending to each value the object's respective membership score. Expressed in general terms, the membership score s_i of an object u_i in value $\{v_l\}$ of set $\mathbf{S}_j$ is given by some membership function $\mu_{\mathbf{S}_j}(x_i\{v_l\})$, where $i = 1, 2, \ldots, n$, $j = 1, 2, \ldots, k$ and $l = 1, 2, \ldots, p$.[15] The value $\{v_l\}$ can be designated by any symbol as long as it is a unique identifier within j. The expression $\mathbf{S}_j\{v_l\}s_i$ is then called a *value-score term* (VST).

Table 2 presents ten hypothetical objects, three condition sets and an outcome set using VSN. $\mathbf{C}_1$ is a bivalent crisp set, $\mathbf{C}_2$ a trivalent crisp set, and $\mathbf{C}_3$ as well as the outcome set $\mathbf{O}$ are bivalent fuzzy sets. For example, the membership score of u_1 in value $v_2 = 1$ of $\mathbf{C}_1$ is given by $\mu_{\mathbf{C}_1}(x_1\{v_2\}) = \mathbf{C}_1\{v_2\}c_1 = \mathbf{C}_1\{1\}0$. In contrast, its score in the same condition but value $v_1 = 0$ is given by $\mu_{\mathbf{C}_1}(x_1\{v_1\}) = \mathbf{C}_1\{v_1\}c_1 = \mathbf{C}_1\{0\}1$. The advantage of bivalent crisp set data is that membership scores for the other value need not be explicitly provided because with only two values, Boolean negation always identifies the membership score in the other value. No degrees of freedom exist. More precisely, for any bivalent crisp set $\mathbf{S}_j$, $\mathbf{S}_j\{v_{l=\mathrm{z}}\}s_i = 1 - \mathbf{S}_j\{v_{l\neq\mathrm{z}}\}s_i$. $\mathbf{C}_1\{0\}1$ thus carries the meaning of applying the logical NOT operator on $\mathbf{C}_1\{1\}1$.[16]

With regards to the bivalent fuzzy set $\mathbf{C}_3$, the membership score of u_1 in value $v_2 = 1$ is given by $\mu_{\mathbf{C}_3}(x_1\{v_2\}) = \mathbf{C}_3\{v_2\}c_1 = \mathbf{C}_3\{1\}0.4$, whereas its membership score in value $v_1 = 0$ is given by $\mu_{\mathbf{C}_3}(x_1\{v_1\}) = \mathbf{C}_3\{v_1\}c_1 = \mathbf{C}_3\{0\}0.6$. As in the case of bivalent crisp sets, membership scores for the other value of a bivalent fuzzy set need not be explicitly provided in Table 2 because two values permit membership score identification by means of Boolean negation.

`TABLE 2 ABOUT HERE`

Finally, the membership score of u_1 in value $v_1 = \alpha$ of condition $\mathbf{C}_2$ is given by $\mu_{\mathbf{C}_2}(x_1\{v_1\}) = \mathbf{C}_2\{v_1\}c_1 = \mathbf{C}_2\{\alpha\}1$. In contrast, its membership scores in values $v_2 = \beta$ and $v_3 = \gamma$ are given by $\mu_{\mathbf{C}_2}(x_1\{\beta,\gamma\}) = \mathbf{C}_2\langle\{\beta\}c_1\{\gamma\}c_1\rangle = \mathbf{C}_2\langle\{\beta\}0\{\gamma\}0\rangle = \mathbf{C}_2\{\beta,\gamma\}0$.[17] No membership scores have to be provided for values v_2 and v_3 because an object can only have membership in one value of a multivalent crisp set. However, unlike bivalent sets which can either be represented by one value or the other through negation, there is no other way of representation for $\mathbf{C}_2$ that is as efficient as that used in Table 2. The reason is the following: if it is known of which value an object is a member, it is also known that it cannot be a member of another value. More precisely, for any bivalent or multivalent crisp set $\mathbf{S}_j$, $\mathbf{S}_j\{v_{l=\mathrm{z}}\}c_i = 1 - (\mathbf{S}_j\{v_1\}c_i + \mathbf{S}_j\{v_2\}c_i + \ldots + \mathbf{S}_j\{v_{l\neq\mathrm{z}}\}c_i + \ldots + \mathbf{S}_j\{v_p\}c_i)$. The opposite logic does not hold because even if the value of which an object is not a member was identified, the information would be insufficient to determine the value of which it was a member. There are exactly $p_j - 1$ degrees of freedom. As a result, more information is needed and the representation of this information can never be more efficient than when the value of membership is used because $p_j - 1$ VSTs would be required.

In summary, if crisp or fuzzy sets are bivalent, or if multivalent sets are crisp, have disjunct values and are represented by membership-identifying value-score terms, all value-specific membership scores are fully determined. With the common notational system of value-scores, a hierarchical representation of subset relations between these three different types of sets becomes possible. It is shown in Figure 2.

FIGURE 2 ABOUT HERE

Bivalent crisp sets are special cases of both bivalent fuzzy sets and multivalent crisp sets on different dimensions. Bivalent crisp sets share with multivalent crisp sets the set of binary membership scores $\mathbb{B} = \{0, 1\}$, and they share with bivalent fuzzy sets the characteristic of having only two values. In contrast, bivalent fuzzy sets and multivalent crisp sets share no commonalities because each possesses a dimension that the other cannot incorporate. With regards to Vink and van Vliet's (2009) doubts about the status of mvQCA, VSN thus illustrates more clearly that mvQCA is a generalization of csQCA and, in consequence, a full-fledged member of the QCA family of comparative configurational techniques.

With complete information on condition sets $\mathbf{C}_1$, $\mathbf{C}_2$, $\mathbf{C}_3$ and the outcome set $\mathbf{O}$, the truth table can be derived. It consists of $d = \prod_{j=1}^{k} p_j$ configurations, where p_j is again the total number of values in set j. The complete truth table of the data presented in Table 2 is shown in Table 3. It lists all $d = 12$ configurations and the associated outcome value (OUT) for each outcome set value of $\mathbf{O}$. Positive configurations are coded as OUT = 1, negative configurations as OUT = 0 and logical remainders as OUT = ?. A configuration's outcome value is determined by the number of strong instances (n) as well as its sufficiency inclusion score (Incl_S).[18] The membership scores of each object in each of the 12 configurations $\mathcal{C}_a$ with $a = 1, 2, \ldots, 12$ are shown in the right part of Table 2. Cells with a gray background indicate the highest membership of a case in the associated configuration that exceeds a value of 0.5. The scalar cardinality $|\mathcal{C}_a|$ of the configuration represented by $\mathcal{C}_a$ is calculated as given in Equation (4) and presented in Table 2.

$$|\mathcal{C}_a| = \sum_{i=1}^{n} \min[\mathbf{C}_1\{v_l\}c_i, \mathbf{C}_2\{v_l\}c_i, \mathbf{C}_3\{v_l\}c_i] \tag{4}$$

The scalar cardinality $|\mathcal{C}_a \cap \mathbf{O}\{v_l\}|$ of each configuration and the respective value of the outcome set $\mathbf{O}\{v_l\}$ is given in Equation (5) and also presented in Table 2.

$$|\mathcal{C}_a \cap \mathbf{O}\{v_l\}| = \sum_{i=1}^{n} \min[\min[\mathbf{C}_1\{v_l\}c_i, \mathbf{C}_2\{v_l\}c_i, \mathbf{C}_3\{v_l\}c_i], \mathbf{O}\{v_l\}o_i] \tag{5}$$

Then, the sufficiency inclusion score of each configuration $\text{Incl}_S(\mathcal{C}_a)$ is the ratio between the two as specified in Equation (6) and presented under column Incl_S in Table 3 for each value of $\mathbf{O}$.

$$\text{Incl}_S(\mathcal{C}_a) = \frac{|\mathcal{C}_a \cap \mathbf{O}\{v_l\}|}{|\mathcal{C}_a|} \tag{6}$$

TABLE 3 ABOUT HERE

All outcome values given in Table 3 should be uncontroversial. No score apart from those indicating perfect inclusion could be considered as qualifying for quasi-sufficiency (e.g. 0.85). The number of cases required to not be coded as a logical remainder is 1. Configuration $\mathcal{C}_{11}$ shows no inclusion score because no object has positive membership in it. The canonical union of fundamental intersections with respect to $\mathbf{O}\{1\}$ is given by set relation (S_2).

$$\overbrace{\mathbf{C}_1\{1\}\mathbf{C}_2\{\gamma\}\mathbf{C}_3\{0\}}^{\mathcal{C}_3} \cup \overbrace{\mathbf{C}_1\{1\}\mathbf{C}_2\{\gamma\}\mathbf{C}_3\{1\}}^{\mathcal{C}_6} \cup \overbrace{\mathbf{C}_1\{0\}\mathbf{C}_2\{\alpha\}\mathbf{C}_3\{0\}}^{\mathcal{C}_7} \subseteq \mathbf{O}\{1\} \qquad (S_2)$$

Using the theorems of Boolean algebra, (S_2) can be reduced to minimal union (S_3).

$$\mathbf{C}_1\{1\}\mathbf{C}_2\{\gamma\} \cup \mathbf{C}_1\{0\}\mathbf{C}_2\{\alpha\}\mathbf{C}_3\{0\} \subseteq \mathbf{O}\{1\} \qquad (S_3)$$

The second value of $\mathbf{C}_1$ in conjunction with the third value of $\mathbf{C}_2$, or the first value of $\mathbf{C}_1$ in conjunction with the first value of $\mathbf{C}_2$ and the first value of $\mathbf{C}_3$ are sufficient for the second value of $\mathbf{O}$.

In summary, if the set type associated with each variant is brought into the standardized system of VSN, the incompatibility between mvQCA and fsQCA disappears. Parameters of fit remain as valid as truth table construction and Boolean minimization procedures. The next section shows how to extend this system further to include multivalent fuzzy sets, which reconcile the two dimensions that are covered separately by bivalent fuzzy sets and multivalent crisp sets. In order to distinguish them from the existing variants, analyses with at least one multivalent fuzzy set will be referred to as generalized-set Qualitative Comparative Analysis (gsQCA).

Generalized-Set QCA

The approach taken by gsQCA builds on *multivalent fuzzy sets*, which represent the generalization of the three existing set types. A multivalent fuzzy set is more general because it combines the value dimension of multivalent crisp sets with the graded-membership dimension of bivalent fuzzy sets. Figure 3 illustrates how a multivalent fuzzy set $\mathbf{S}_j^*$ should be conceived of geometrically.

FIGURE 3 ABOUT HERE

Three objects are shown, each represented by a different plot marker. A diamond designates u_1, a square u_2 and a circle u_3. The first three values of $\mathbf{S}_j^*$ form a three-dimensional vector space in the unit interval. In contrast to multivalent crisp sets, under whose structure objects are restricted to membership in exactly one value and non-membership in all other values, multivalent fuzzy sets permit objects to possess membership in any number of values to any degree. For example, u_1 can only be distinguished by its notation from an object in a multivalent crisp set because it has full membership in v_1, but no membership in v_2 and v_3 of $\mathbf{S}_j^*$. In contrast, u_2 has no membership in v_1, but it is a full member of v_2 and v_3. Object u_3 is most representative of a multivalent fuzzy set, with a partial membership of 0.5 in each of the three values. These relations can be summarized in VSN as u_1: $\mathbf{S}_j^*\langle\{v_1\}1.0\{v_2\}0.0\{v_3\}0.0\rangle$, u_2: $\mathbf{S}_j^*\langle\{v_1\}0.0\{v_2\}1.0\{v_3\}1.0\rangle$ and u_3: $\mathbf{S}_j^*\langle\{v_1\}0.5\{v_2\}0.5\{v_3\}0.5\rangle$.[19] An example from the democratization literature serves to illustrate the advantages of multivalent fuzzy sets for empirical research.

The study by Hartmann and Kemmerzell (2010) analyses the provision and implementation of party bans in sub-Saharan Africa between 1990 and 2007. One condition the authors deem important is the "colonial background" (**CB**) of a country. It comprises three values: "British" (B), "French" (F) and "Other" (O).[20] It is argued that countries with a British legacy would have a more tolerant party system than countries which had been exposed to French political culture. Unfortunately, the authors do not provide a formal definition of when a country is characterized by one or the other background because several countries have experienced both British and French colonial rule. The case of Mauritius warrants closer attention in this respect.[21]

The French ruled Mauritius from 1715 to 1810, the British from 1810 to the year of independence in 1968, but is the later and much longer period of British rule sufficient enough a criterion for considering the island as fully out of the set of countries with a French colonial legacy? The empirics of the case suggest a negative answer with qualifications. On the one hand, English is the official language, but it is rarely spoken outside of government business (Gall and Gleason, 2012:472), trade relations with France have remained about as close as those with the UK (International Monetary Fund, 2011:684), and French Codes have prevailed over British Law in many areas of the country's legal system (Brendon, 2007:96).[22] Mauritius' close ties with France, which were further strengthened after independence, also allowed the country to become the first member of the Commonwealth to be associated with the European Economic Community in 1972, a year before the UK itself joined the Common Market (Houbert, 1981:89f.). Although 'the Union Jack waved over Mauritius for 160 years, the island never effectively became British.' (Tinker, 1977:323).

On the other hand, not only have many of the country's political leaders been influenced by British Fabian rather than French socialism (Dommen and Dommen, 1999:88-90; Meisenhelder, 1997:281), but the Mauritian electoral system is also of a modified Westminster style, even though the party landscape is highly fragmented and party discipline is practically non-existent (Srebrnik, 2002:279). From this concise juxtaposition of French and English influences, an exclusive assignment to any one single value of **CB** appears unreasonably crude.

Multivalent fuzzy sets solve the problem of inaccurate assignments induced by the use of multivalent crisp or bivalent fuzzy sets. If it is accepted that Mauritius' colonial experience is a 'conjunction of British and French cultures' (Dommen and Dommen, 1999:75), then it could be regarded as a partial member of both the set of countries with a French colonial legacy and that with a British legacy. In VSN, this could, for instance, be expressed as MU: $\mathbf{CB}\langle\{B\}0.5\{F\}0.5\{O\}0\rangle$. While multiple partial membership is important, independent criteria for determining the degree of membership are still missing.

Independence between values is essential. For example, consider what would happen if membership across all values were to be limited to unity. If $\mathbf{CB}\{F\}$ had been defined as at least 120 years of uninterrupted rule by France over the period 1715-1950, and Mauritius would have been first assigned to $\mathbf{CB}\{F\}$, then $\mathbf{CB}\{B\}$ would have had to be defined as at most 25 years of uninterrupted rule by Britain over the period 1715-1950 if membership unity had had to be ensured. Conversely, if $\mathbf{CB}\{B\}$ had been defined as at least 120 years of uninterrupted rule by Britain over the period 1715-1950, and Mauritius would have been first assigned to $\mathbf{CB}\{B\}$, then $\mathbf{CB}\{F\}$ would have had to be defined as at most zero years of uninterrupted rule by France over the period 1715-1950.[23]

For this reason, membership in multivalent fuzzy sets need not sum up to unity across values. The assignment of an object to a value must be independent from the assignment of the same object to all other values, both spatially and temporally. For example, if Hartmann and Kemmerzell (2010) had stipulated that at least 120 years of uninterrupted rule by a foreign power over the period from 1715 to 1950 be required to assign any colonial legacy to a country, then Mauritius would have been coded as MU: $\mathbf{CB}\langle\{B\}1\{F\}0.79\{O\}0\rangle$ in value-score notation.[24]

FIGURE 4 ABOUT HERE

The concept of the multivalent fuzzy set has several implications, from the perspective of set-theoretic principles in general, and for QCA in particular. Boolean negation does not apply to multivalent fuzzy sets because an object's assignment to any value of such a set is unrelated to its assignment to all other values. The notational representation under a multivalent fuzzy set thus has to include all VSTs. Figure 4 illustrates the extended hierarchy of set types. The addition of multivalent fuzzy sets includes as direct special cases both bivalent fuzzy sets and multivalent crisp sets, and indirectly also bivalent crisp sets. As such, multivalent fuzzy sets break down the dimensional incompatibility between multiple values and graded membership. In the remainder of this article, a generic example illustrates how multivalent sets can be applied in performing QCA. If the data contains at least one multivalent fuzzy set, the application of QCA will be referred to as gsQCA.

A new data table incorporating the multivalent fuzzy set $\mathbf{C}_2^*$ is presented in the left part of Table 4. This condition set comprises three values: $\{a\}$, $\{b\}$ and $\{c\}$. All other sets remain the same.

`TABLE 4 ABOUT HERE`

The complete truth table that results from applying the truth table algorithm used in the preceding section to the set data in the left part of Table 4 is shown in Table 5. Notice that, on the one hand, some objects have membership above 0.5 in more than one configuration as indicated by more than one gray-colored cell in each row of Table 4 and the multiple appearance of the same index in column u_i of Table 5. For example, u_5 is a strong instance of configurations $\mathcal{C}_2$, $\mathcal{C}_4$ and $\mathcal{C}_6$. On the other hand, no configuration features u_8 as a strong instance. Generally, if an object's membership does not exceed 0.5 under at least one value of a multivalent fuzzy set, it will not have strong membership in any truth table configuration. Thus, with multivalent fuzzy sets, not only does Boolean negation not help in identifying other value membership scores, but also can objects be a strong instance of more than one configuration or even none at all.

TABLE 5 ABOUT HERE

In order to examine the implications of gsQCA further, consider a decrease in the cut-off for the sufficiency inclusion score to 0.85. The canonical union for all positive configurations with respect to $\mathbf{O}\{1\}$ is then given by set relation (S_4).

$$\overbrace{\mathbf{C}_1\{1\}\mathbf{C}_2^*\{a\}\mathbf{C}_3\{0\}}^{\mathcal{C}_1} \cup \overbrace{\mathbf{C}_1\{1\}\mathbf{C}_2^*\{b\}\mathbf{C}_3\{0\}}^{\mathcal{C}_3} \cup \overbrace{\mathbf{C}_1\{1\}\mathbf{C}_2^*\{a\}\mathbf{C}_3\{1\}}^{\mathcal{C}_4} \cup \overbrace{\mathbf{C}_1\{1\}\mathbf{C}_2^*\{c\}\mathbf{C}_3\{0\}}^{\mathcal{C}_5} \subseteq \mathbf{O}\{1\} \quad (S_4)$$

Minimization of these four fundamental intersections yields the minimal union with two prime implicants given in set relation (S_5). Condition set $\mathbf{C}_2^*$ is redundant in the first, second and fourth, and condition set $\mathbf{C}_3$ is redundant in the first and third fundamental intersections.

$$\mathbf{C}_1\{1\}\mathbf{C}_2^*\{a\} \cup \mathbf{C}_1\{1\}\mathbf{C}_3\{0\} \subseteq \mathbf{O}\{1\} \quad (S_5)$$

If, however, the cut-off for the sufficiency inclusion score is raised to 0.9, $\mathcal{C}_4$ drops out and the reduced canonical union for the remaining configurations is now given by set relation (S_6).

$$\overbrace{\mathbf{C}_1\{1\}\mathbf{C}_2^*\{a\}\mathbf{C}_3\{0\}}^{\mathcal{C}_1} \cup \overbrace{\mathbf{C}_1\{1\}\mathbf{C}_2^*\{b\}\mathbf{C}_3\{0\}}^{\mathcal{C}_3} \cup \overbrace{\mathbf{C}_1\{1\}\mathbf{C}_2^*\{c\}\mathbf{C}_3\{0\}}^{\mathcal{C}_5} \subseteq \mathbf{O}\{1\} \quad (S_6)$$

Condition set $\mathbf{C}_2^*$ is again redundant across these three fundamental products. The resulting minimal union is given by set relation (S_7).

$$\mathbf{C}_1\{1\}\mathbf{C}_3\{0\} \subseteq \mathbf{O}\{1\} \tag{S_7}$$

The minimal union consists of a single prime implicant that covers three configurations: $\mathcal{C}_1$, $\mathcal{C}_3$ and $\mathcal{C}_5$. However, these configurations only contain u_{10} as their single strong instance. It is the only object that has membership above 0.5 in all configurations. With multivalent fuzzy sets, complex solutions therefore need not cover at least as many empirically strong instances as there have been fundamental intersections in the canonical union.

Conclusions

This article set out to demonstrate that the strong emphasis on the differences between csQCA, mvQCA and fsQCA which has existed so far in the literature on comparative configurational methodology can be turned into an argument about their commonalities. The approach whereby this is made possible has been referred to as generalized-set QCA (gsQCA). The core concept underlying gsQCA is the multivalent fuzzy set. This set type unites the value dimension of multivalent crisp sets and the membership dimension of bivalent fuzzy sets such that all three existing QCA variants become special cases of gsQCA.

In gsQCA, at least one condition set in the data has more than two values, for each of which a membership function maps cases into the unit interval $\mathbb{U} = [0, 1]$. No restrictions with regards to the number of values or the membership score an object can be assigned to under each value exist. In contrast, if at least one condition set in the data has two values, for each of which a membership function maps objects into the unit interval $\mathbb{U} = [0, 1]$, and all remaining condition sets have two values, for each of which a characteristic function maps objects into the binary set $\mathbb{B} = \{0, 1\}$, but to only one of which objects can be assigned in each of the two condition set types, gsQCA reduces to fsQCA. If at least one condition set has more than two values and all remaining condition sets have two values, for each of which a characteristic function maps objects into the binary set $\mathbb{B} = \{0, 1\}$, but to only one of which objects can be assigned in each of the two condition set types, gsQCA becomes mvQCA. Finally, if all condition sets have two values, for each of which a characteristic function maps objects into the binary set $\mathbb{B} = \{0, 1\}$, and to only one of which objects can be assigned, gsQCA collapses to csQCA.

The introduction of gsQCA opens up a unifying perspective on the QCA family of configurational comparative methods. Most importantly, the three existing variants should not be seen any longer as standing in a competitive relation, but as special cases of a general approach for the analysis of specific combinations of set types. No disadvantages result from this generalization. In gsQCA, all parameters of fit, truth table construction and Boolean minimization procedures, as well as the derivation of complex, intermediate and parsimonious solutions, remain wholly applicable. If there is a cost to be found, it is the more elaborate system of value-score notation which is required. In consideration of the added value of the approach, which has been primarily theoretical rather than empirical within the scope of this article, small additions to data coding should be but a symbolic price to pay.

Notes

[1]Rihoux and Ragin (2009:xix) define “configurational comparative methods” to include all variants of QCA as well as some of Mill’s methods. Coincidence Analysis (Baumgartner, 2009, 2013) is another recent addition to this family of methods.

[2]According to this theorem, an object can have a membership score of at most 0.5 in more than one truth table configuration, but it can only have membership above 0.5 in a single configuration. For a formal proof, see Mendel and Korjani (2012).

[3]The first application of csQCA has been presented by Ragin, Mayer and Drass (1984).

[4]The former is known as the “law of excluded middle”, the latter as the “law of contradiction”. For simplicity, the intersection operator “$\cap$” will be dropped from further set-theoretic expressions.

[5]“Boolean absorption”, “Boolean reduction” and “Boolean elimination” are alternative terms.

[6]Proofs:

$$\begin{aligned} \mathbf{S}_a\mathbf{S}_b \cup \mathbf{s}_a\mathbf{S}_b &= \mathbf{S}_b(\mathbf{S}_a \cup \mathbf{s}_a) && \text{by } (A_4) \\ &= \mathbf{S}_b && \text{by } (A_2) \end{aligned}$$

$$\begin{aligned} \mathbf{S}_a\mathbf{S}_b \cup \mathbf{s}_a\mathbf{S}_c \cup \mathbf{S}_b\mathbf{S}_c &= \mathbf{S}_a\mathbf{S}_b \cup \mathbf{s}_a\mathbf{S}_c \cup (\mathbf{S}_a \cup \mathbf{s}_a)\mathbf{S}_b\mathbf{S}_c && \text{by } (A_4) \\ &= \mathbf{S}_a\mathbf{S}_b \cup \mathbf{s}_a\mathbf{S}_c \cup \mathbf{S}_a\mathbf{S}_b\mathbf{S}_c \cup \mathbf{s}_a\mathbf{S}_b\mathbf{S}_c && \text{by } (A_2) \\ &= \mathbf{S}_a\mathbf{S}_b \cup \mathbf{S}_a\mathbf{S}_b\mathbf{S}_c \cup \mathbf{s}_a\mathbf{S}_c \cup \mathbf{s}_a\mathbf{S}_b\mathbf{S}_c && \text{by } (A_1) \\ &= \mathbf{S}_a\mathbf{S}_b(1 \cup \mathbf{S}_c) \cup \mathbf{s}_a\mathbf{S}_c(1 \cup \mathbf{S}_b) && \text{by } (A_2), (A_3) \\ &= \mathbf{S}_a\mathbf{S}_b \cup \mathbf{s}_a\mathbf{S}_c && \text{by } \mathbf{S}_j + 1 = 1 \end{aligned}$$

[7]The same result can also be derived algebraically in four steps.

$$\begin{aligned} \overbrace{\mathbf{C}_1\mathbf{C}_2\mathbf{C}_3}^{\mathcal{C}_1} \cup \overbrace{\mathbf{C}_1\mathbf{C}_2\mathbf{c}_3}^{\mathcal{C}_2} \cup \overbrace{\mathbf{C}_1\mathbf{c}_2\mathbf{C}_3}^{\mathcal{C}_3} \cup \overbrace{\mathbf{C}_1\mathbf{c}_2\mathbf{c}_3}^{\mathcal{C}_4} &\subseteq \mathbf{O} \\ (\mathbf{C}_1\mathbf{C}_2)(\mathbf{C}_3 \cup \mathbf{c}_3) \cup (\mathbf{C}_1\mathbf{c}_2)(\mathbf{C}_3 \cup \mathbf{c}_3) &\subseteq \\ (\mathbf{C}_1\mathbf{C}_2) \cup (\mathbf{C}_1\mathbf{c}_2) &\subseteq \\ \mathbf{C}_1(\mathbf{C}_2 \cup \mathbf{c}_2) &\subseteq \\ \mathbf{C}_1 &\subseteq \end{aligned}$$

[8]See also the special issue in this journal (Ragin and Pennings, 2005).

[9]The laws of excluded middle and contradiction do not hold any longer under these assumptions.

[10]For more detailed treatments of membership functions, see Klir, St. Clair and Yuan (1997:83-86), Clark et al. (2008:37-45) or Buckley and Eslami (2002:55-62).

[11]For example, Hartmann and Kemmerzell (2010:645) 'are fully aware of the criticisms raised against MV-QCA'.

[12]Potentially, multi-value outcome sets are also possible, but they cannot be processed yet by any of the above-mentioned software programs. While Tosmana accepts multi-value outcome sets, these have to be dichotomized manually before minimization.

[13]For example, a multivalent crisp set with three values is a trivalent crisp set.

[14]In csQCA and fsQCA, these two values are often referred to as "presence" and "absence".

[15]In the remainder of this article, the term "membership function" also includes characteristic functions.

[16]The logical NOT is usually indicated in csQCA and fsQCA either by lower-case letters, in contrast to upper-case letters, by a tilde sign preceding the set name or a prime. These indicators do not apply in value-score notation.

[17]Greek letters have been used for $\mathbf{C}_2$ in order to emphasize that the identifier of a value need not be numeric. Angled brackets are used when more than one value-score term is provided for a case within a single expression.

[18]*Inclusion* corresponds to what Ragin (2006) refers to as *consistency*.

[19]These expressions may be collapsed to u_1: $\mathbf{S}_j\langle\{v_1\}1.0\{v_2, v_3\}0.0\rangle$, u_2: $\mathbf{S}_j\langle\{v_1\}0.0\{v_2, v_3\}1.0\rangle$ and u_3: $\mathbf{S}_j\{v_1, v_2, v_3\}0.5$.

[20]The original values are "British" (2), "French" (1) and "Other" (0).

[21]Correspondence with the authors confirmed several coding typos in the truth table on page 652. Botswana's, Mauritius', South Africa's and Zimbabwe's correct colonial background is "British" (2).

[22]Creole, a French patois, is spoken by about 80% of the population. In 2010, trade with France amounted to US$773.8m, with the UK to US$521.6m. French Law includes the Civil Code, the Penal Code, the Code of Commerce and the Code of Civil Procedure, English Law the Bankruptcy Law, the Company Law, the Law of Evidence and the Law of Criminal Procedure (Central Office of Information for British Information Services, 1968:12).

[23]$1 = 95/120 + 25/120$ and $1 = 120/120 + 0/120$.

[24]Ninety-five years of French rule divided by 120 years of rule for full membership yield a partial membership of 0.79. As the British ruled Mauritius for 158 years, the last 38 years represent irrelevant variation beyond full set membership with respect to the definition.

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Table 1: Hypothetical Truth Table

$\mathcal{C}_a$	$\mathbf{C}_1$	$\mathbf{C}_2$	$\mathbf{C}_3$	OUT	n
1	1	1	1	1	≥ 1
2	1	1	0	1	≥ 1
3	1	0	1	1	≥ 1
4	1	0	0	1	≥ 1
5	0	1	1	0	≥ 1
6	0	1	0	C	≥ 2
7	0	0	1	?	0
8	0	0	0	?	0

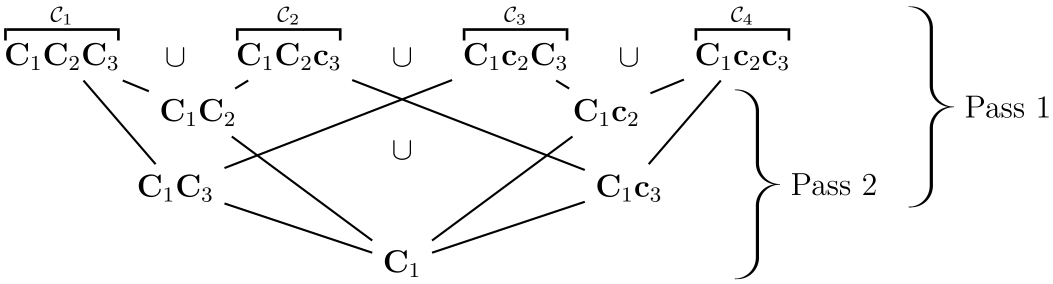

Figure 1: Graphical minimization of Set Relation (S_1)

Table 2: Set-Value and Configuration Membership in Value-Score Notation

	Set-Value Membership				Configuration Membership in $\mathcal{C}_{a\,=\,\dots}$ (Table 3)											
u_i	$\mathbf{C}_1$	$\mathbf{C}_2$	$\mathbf{C}_3$	$\mathbf{O}$	1	2	3	4	5	6	7	8	9	10	11	12
1	$\{1\}0$	$\{\alpha\}1$	$\{1\}0.4$	$\{1\}0.7$	0.0	0.0	0.0	0.0	0.0	0.0	0.6	0.0	0.0	0.4	0.0	0.0
2	$\{1\}0$	$\{\gamma\}1$	$\{1\}0.2$	$\{1\}0.1$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.8	0.0	0.0	0.2
3	$\{1\}1$	$\{\beta\}1$	$\{1\}0.8$	$\{1\}0.5$	0.0	0.8	0.0	0.0	0.2	0.0	0.0	0.0	0.0	0.0	0.0	0.0
4	$\{1\}1$	$\{\beta\}1$	$\{1\}1.0$	$\{1\}0.2$	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
5	$\{1\}1$	$\{\gamma\}1$	$\{1\}0.6$	$\{1\}1.0$	0.0	0.0	0.4	0.0	0.0	0.6	0.0	0.0	0.0	0.0	0.0	0.0
6	$\{1\}0$	$\{\gamma\}1$	$\{1\}0.1$	$\{1\}0.9$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.9	0.0	0.0	0.1
7	$\{1\}1$	$\{\alpha\}1$	$\{1\}0.7$	$\{1\}0.3$	0.3	0.0	0.0	0.7	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
8	$\{1\}1$	$\{\alpha\}1$	$\{1\}0.9$	$\{1\}0.4$	0.1	0.0	0.0	0.9	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
9	$\{1\}0$	$\{\beta\}1$	$\{1\}1.0$	$\{1\}0.0$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0
10	$\{1\}1$	$\{\gamma\}1$	$\{1\}0.0$	$\{1\}1.0$	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
				$\|\mathcal{C}_a\|$	0.4	1.8	1.4	1.6	0.2	0.6	0.6	1.0	1.7	0.4	0.0	0.3
				$\|\mathcal{C}_a \cap \mathbf{O}\{1\}\|$	0.4	0.7	1.4	0.7	0.2	0.6	0.6	0.0	1.0	0.4	0.0	0.2
				$\|\mathcal{C}_a \cap \mathbf{O}\{0\}\|$	0.4	1.3	0.4	0.4	0.2	0.4	0.3	1.0	0.9	0.3	0.0	0.3

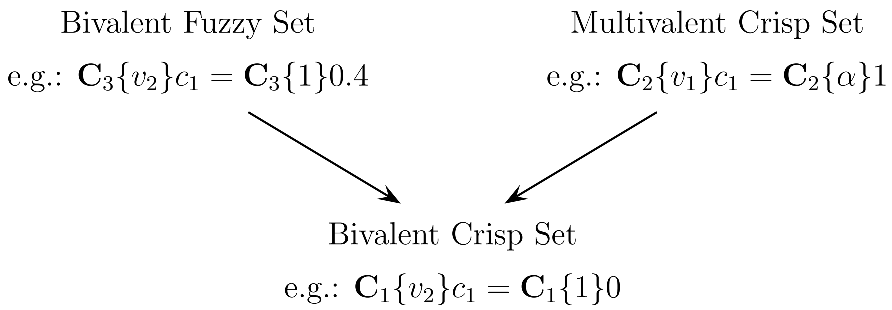

Figure 2: Hierarchy of Set Types

Table 3: Truth Table

	Set-Values				Incl_S		OUT [a]		
$\mathcal{C}_a$	$\mathbf{C}_1$	$\mathbf{C}_2$	$\mathbf{C}_3$	n	$\mathbf{O}\{1\}$	$\mathbf{O}\{0\}$	$\mathbf{O}\{1\}$	$\mathbf{O}\{0\}$	u_i
1	1	α	0	0	1.00	1.00	?	?	-
2	1	β	1	2	0.39	0.72	0	0	3, 4
3	1	γ	0	1	1.00	0.29	1	0	10
4	1	α	1	2	0.44	0.25	0	0	7, 8
5	1	β	0	0	1.00	1.00	?	?	-
6	1	γ	1	1	1.00	0.67	1	0	5
7	0	α	0	1	1.00	0.50	1	0	1
8	0	β	1	1	0.00	1.00	0	1	9
9	0	γ	0	2	0.59	0.53	0	0	2, 6
10	0	α	1	0	1.00	0.75	?	?	-
11	0	β	0	0	-	-	?	?	-
12	0	γ	1	0	0.67	1.00	?	?	-

[a] number of cases cut-off: 1; inclusion cut-off: 0.9

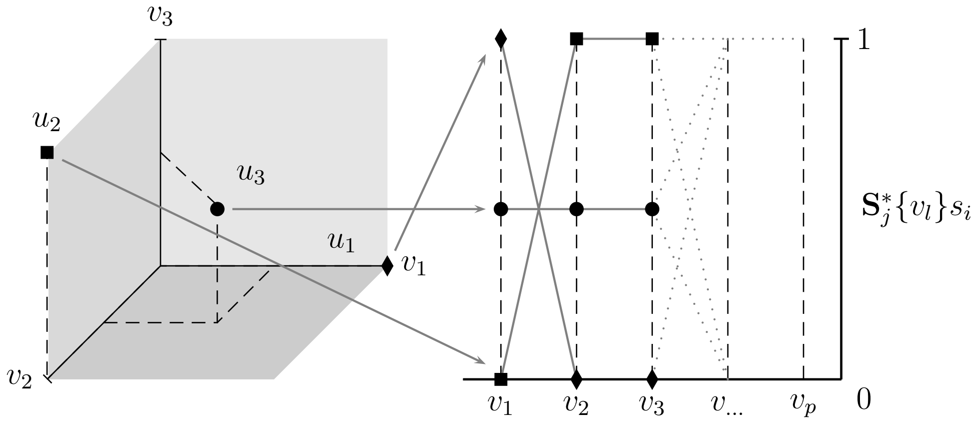

Figure 3: Geometric Representation of a Multivalent Fuzzy Set

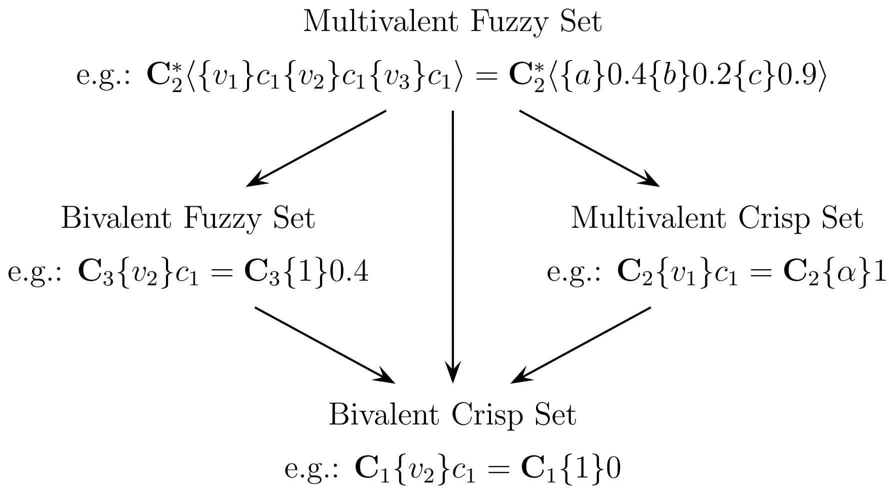

Figure 4: Extended Hierarchy of Set Types

Table 4: Set-Value and Configuration Membership, with Multivalent Fuzzy Set

	Set-Value Membership				Configuration $\mathcal{C}_{a=\dots}$ (Table 5)											
u_i	$\mathbf{C}_1$	$\langle\ \mathbf{C}_2^*\ \rangle$	$\mathbf{C}_3$	$\mathbf{O}$	1	2	3	4	5	6	7	8	9	10	11	12
1	$\{1\}0$	$\{a\}0.1\{b\}0.2\{c\}0.7$	$\{1\}0.4$	$\{1\}0.7$	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.2	0.6	0.1	0.2	0.4
2	$\{1\}0$	$\{a\}0.7\{b\}0.1\{c\}0.4$	$\{1\}0.2$	$\{1\}0.1$	0.0	0.0	0.0	0.0	0.0	0.0	0.7	0.1	0.4	0.2	0.1	0.2
3	$\{1\}1$	$\{a\}0.5\{b\}0.7\{c\}0.1$	$\{1\}0.8$	$\{1\}0.5$	0.2	0.7	0.1	0.5	0.2	0.1	0.0	0.0	0.0	0.0	0.0	0.0
4	$\{1\}1$	$\{a\}0.1\{b\}0.4\{c\}0.8$	$\{1\}1.0$	$\{1\}0.2$	0.0	0.4	0.0	0.1	0.0	0.8	0.0	0.0	0.0	0.0	0.0	0.0
5	$\{1\}1$	$\{a\}0.9\{b\}1.0\{c\}0.8$	$\{1\}0.6$	$\{1\}1.0$	0.4	0.6	0.4	0.6	0.4	0.6	0.0	0.0	0.0	0.0	0.0	0.0
6	$\{1\}0$	$\{a\}0.7\{b\}0.3\{c\}0.3$	$\{1\}0.1$	$\{1\}0.9$	0.0	0.0	0.0	0.0	0.0	0.0	0.7	0.1	0.3	0.1	0.3	0.1
7	$\{1\}1$	$\{a\}0.6\{b\}0.9\{c\}0.8$	$\{1\}0.7$	$\{1\}0.3$	0.3	0.7	0.3	0.6	0.3	0.7	0.0	0.0	0.0	0.0	0.0	0.0
8	$\{1\}1$	$\{a\}0.3\{b\}0.1\{c\}0.4$	$\{1\}0.9$	$\{1\}0.4$	0.1	0.1	0.1	0.3	0.1	0.4	0.0	0.0	0.0	0.0	0.0	0.0
9	$\{1\}0$	$\{a\}0.8\{b\}0.2\{c\}0.8$	$\{1\}1.0$	$\{1\}0.0$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.2	0.0	0.8	0.0	0.8
10	$\{1\}1$	$\{a\}0.7\{b\}0.9\{c\}0.6$	$\{1\}0.0$	$\{1\}1.0$	0.7	0.0	0.6	0.0	0.9	0.0	0.0	0.0	0.0	0.0	0.0	0.0
				$\lvert\mathcal{C}_a\rvert$	1.7	2.5	1.5	2.1	1.9	2.6	1.5	0.6	1.3	1.2	0.6	1.5
				$\lvert\mathcal{C}_a \cap \mathbf{O}\{1\}\rvert$	1.7	1.7	1.5	1.8	1.9	1.6	0.9	0.4	1.0	0.3	0.6	0.6
				$\lvert\mathcal{C}_a \cap \mathbf{O}\{0\}\rvert$	0.6	1.7	0.5	1.5	0.6	2.0	0.9	0.6	0.8	1.2	0.4	1.4

Table 5: Truth Table

	Set-Values				Incl_S		OUT [a]		
$\mathcal{C}_a$	$\mathbf{C}_1$	$\mathbf{C}_2^*$	$\mathbf{C}_3$	n	$\mathbf{O}\{1\}$	$\mathbf{O}\{0\}$	$\mathbf{O}\{1\}$	$\mathbf{O}\{0\}$	u_i
1	1	a	0	1	1.00	0.35	1	0	10
2	1	b	1	3	0.68	0.68	0	0	3, 5, 7
3	1	c	0	1	1.00	0.33	1	0	10
4	1	a	1	1	0.86	0.71	1/0	0	5, 7
5	1	b	0	1	1.00	0.32	1	0	10
6	1	c	1	3	0.62	0.77	0	0	4, 5, 7
7	0	a	0	2	0.60	0.60	0	0	2, 6
8	0	b	1	0	0.67	1.00	?	?	-
9	0	c	0	1	0.77	0.62	0	0	1
10	0	a	1	1	0.25	1.00	0	1	9
11	0	b	0	0	1.00	0.67	?	?	-
12	0	c	1	1	0.40	0.93	0	1	9

[a] number of cases cut-off: 1; inclusion cut-off: 0.85/0.90